A New Finite Element Model for Reduced Order Electromagnetic Modeling

Yu Zhu, *Student Member, IEEE,* and Andreas C. Cangellaris, *Fellow, IEEE*

***Abstract*—This paper introduces a new formulation suitable for direct model order reduction of finite element approximations of electromagnetic systems using Krylov subspace methods. The proposed formulation utilizes a finite element model of Maxwell's curl equations to generate a state-space representation of the electromagnetic system most suitable for the implementation of model order reduction techniques based on Krylov subspaces. It is shown that, with a proper selection of the finite element interpolation functions for the fields, the proposed formulation is equivalent to the commonly used approximation of the vector wave equation with tangentially continuous vector finite elements. This equivalence is exploited to improve the computational efficiency of the model order reduction process.**

***Index Terms*—FEM formulation, model order reduction.**

I. Introduction

Asympotic waveform evaluation (AWE) is popular in FEM modeling for fast frequency sweep and macro-modeling of electromagnetic components [1], [2]. However, because of lack of numerical robustness [3], its bandwidth of accuracy does not improve with increasing the order of the Pade approximation effected by AWE. Therefore, AWE has to be performed on multiple expansion frequencies for broadband response. Because of this deficiency of the AWE process, more efficient and robust model order reduction techniques based on Krylov subspaces have been the topic of significant research, primarily in the area of very large circuit simulation [3], [4]. The encouraging results from the application of Krylov subspace-based model order reduction in large circuit simulation have prompted interest for their adoption and application for the reduction of finite element approximations of general electromagnetic problems. One of the primary obstacles in this effort has been the lack of a suitable state-space representation of the discrete electromagnetic system. To explain, the most common FEM formulation for the electromagnetic problem is the one based on the vector wave equation, and its matrix form for the general case of lossy media and unbounded domains exhibits both linear and quadratic frequency dependence. This limits the direct application of Krylov subspace methods for model order reduction of such FEM models to bounded lossless domains where the linear dependence on frequency is absent [5].

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The authors are with the Department of Electrical and Computer Engineering, University of Illinois at Urbana-Champaign, Urbana, IL 61801 USA (e-mail: cangella@uiuc.edu).

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As clearly demonstrated in [6], discrete approximations of electromagnetic problems based on Maxwell's curl equations are directly compatible with Krylov subspace model order reduction methods. The major disadvantage of such a formulation is the doubling of the size of the discrete problem, since now both electric and magnetic fields are involved in the vector of unknowns. This impacts the computational efficiency of the iterative process involved in the generation of the Krylov subspace. A way to circumvent this difficulty is presented in this paper.

II. Proposed FEM Model

The development begins by considering Maxwell's equations in the Laplace domain,

$$\nabla \times \vec{E} = -s\vec{B} \qquad \nabla \times \left(\vec{B}/\mu\right) = s\epsilon\vec{E} + \sigma\vec{E} \tag{1}$$

where $s = j\omega$. The discrete form of the equations is obtained through the discretization of the computational domain, and the expansion of $\vec{E}$ and $\vec{B}$ in the tangentially-continuous vector (TV) space W_t, and the normally-continuous vector (NV) space W_n, respectively [7].

Let $\vec{w}_t$ denote a basis function of W_t and $\vec{w}_n$ a basis function of W_n. Application of Galerkin's method, where the curl equations (1) are multiplied by $\vec{w}_n$ and $\vec{w}_t$, respectively, and integrated over the domain of interest, yields

$$\begin{aligned} &\int_\Omega \nabla \times \vec{E} \cdot \frac{1}{\mu}\vec{w}_n\, dv \\ &\quad = -s\int_\Omega \vec{w}_n \cdot \frac{1}{\mu}\vec{B}\, dv \\ &\int_\Omega \nabla \times \vec{w}_t \cdot \frac{1}{\mu}\vec{B}\, dv + \oint_{S_0+S_1} \hat{n} \times \vec{H} \cdot \vec{w}_t\, ds \\ &\quad = s\int_\Omega \vec{w}_t \cdot \epsilon\vec{E}\, dv + \int_\Omega \vec{w}_t \cdot \sigma\vec{E}\, dv \end{aligned} \tag{2}$$

where $\hat{n}$ points out of the computational domain.

S_0 is the surface of physical ports that will be used for the multi-port representation of the system. Let us assume, for the sake of simplicity, that S_0 involves only one port and its modes are known. Thus, $\hat{n} \times \vec{H}$ can be expanded as

$$\hat{n} \times \vec{H} = \sum_{n=0}^{p-1} i_n \vec{h}_n(S_0) \tag{3}$$

where i_n is the expansion coefficient and $\vec{h}_n(S_0)$ is the tangential magnetic field of the nth mode on S_0.

S_1 is the truncation surface of the unbounded portion of the domain. For our purposes, a first-order absorbing boundary condition (ABC) is imposed on S_1

$$\hat{n} \times \vec{H} = \frac{1}{\eta}\, \hat{n} \times \hat{n} \times \vec{E} \tag{4}$$

where η is the surface impedance. From these equations, the matrix form of the FEM approximation is obtained as

$$\begin{pmatrix} 0 & D \\ -D^T & Z \end{pmatrix} \begin{pmatrix} b \\ e \end{pmatrix} = -s \begin{pmatrix} P_b & 0 \\ 0 & P_e \end{pmatrix} \begin{pmatrix} b \\ e \end{pmatrix} + \begin{pmatrix} 0 \\ B \end{pmatrix} I(s) \tag{5}$$

where the vectors e and b contain the expansion coefficients of $\vec{E}$ and $\vec{B}$, respectively, $I(s)$ contains the coefficients i_n, and

$$\begin{aligned} D_{ij} &= \int_\Omega \nabla \times \vec{w}_{t,\,i} \cdot \frac{1}{\mu}\, \vec{w}_{n,\,j}\, dv, \\ P_{b,\,ij} &= \int_\Omega \vec{w}_{n,\,i} \cdot \frac{1}{\mu}\, \vec{w}_{n,\,j}\, dv \\ B_{ij} &= \int_{S_0} \hat{n} \times \vec{w}_{t,\,i} \cdot \hat{n} \times \vec{h}_j\, ds, \\ P_{e,\,ij} &= \int_\Omega \vec{w}_{t,\,i} \cdot \epsilon \vec{w}_{t,\,j}\, dv \\ Z_{ij} &= \int_\Omega \vec{w}_{t,\,i} \cdot \sigma \vec{w}_{t,\,j}\, dv + \int_{S_1} \hat{n} \times \vec{w}_{t,\,i} \cdot \frac{1}{\eta}\, \hat{n} \times \vec{w}_{t,\,j}\, ds. \end{aligned} \tag{6}$$

In compact form, the discrete system of (5) is given by

$$(G + sC)X = FI(s) \tag{7}$$

where $X = (b^T, e^T)^T$ and

$$G = \begin{pmatrix} 0 & D \\ -D^T & Z \end{pmatrix}, \quad C = \begin{pmatrix} P_b & 0 \\ 0 & P_e \end{pmatrix}, \quad F = \begin{pmatrix} 0 \\ B \end{pmatrix}. \tag{8}$$

Similarly, the tangential electrical field $\vec{E}$ on S_0 is also expanded in terms of the electric mode functions $\vec{e}_n(S_0)$

$$\vec{E} = \sum_{n=0}^{p-1} v_n \vec{e}_n(S_0). \tag{9}$$

The desired outputs are the expansion coefficients v_n for the tangential electric field on S_0. In matrix form, the vector $V(s)$ of these coefficients is obtained from the solution of (7) as

$$V(s) = L^H (G + sC)^{-1} FI(s) = H(s)I(s) \tag{10}$$

where the elements of the matrix L are given by

$$L_{ij} = \int_{S_0} \hat{n} \times \vec{w}_{t,\,i} \cdot \hat{n} \times \vec{e}_j\, ds. \tag{11}$$

Equation (10) defines a generalized mode impedance matrix that captures the electromagnetic properties of the structure inside the computational domain through a global impedance relationship between the tangential magnetic and electric fields on the ports S_0.

The form of (10) exhibits linear frequency dependency and thus is directly compatible with model order reduction algorithms based on Krylov subspaces. For example, the PRIMA algorithm of [4] could be used. However, in the process of generating the orthonormal base of Krylov subspace [6], $Kr(A, R, n)$, where $A = -(G + s_0 C)^{-1} C$, $R = (G + s_0 C)^{-1} F$, s_0 is expansion complex frequency, and n is order of the reduced-order model, the LU factorization of the matrix $(G + s_0 C)$ is involved. Considering that both electric and magnetic fields are solved for in this formulation, the dimension of this matrix is twice that of a vector finite element approximation of the vector wave equation for the electric field. So, it appears that the ability to handle unbounded regions and lossy media in a way compatible with Krylov subspace model order reduction methods comes at the expense of increased computational cost. It is shown next that this extra computational cost can be avoided.

III. Equivalence of Two Weak Formulations

In this section it is shown that the FEM formulation given in (5) is equivalent to the one associated with the weak form of the vector wave equation

$$(S + sZ + s^2 P_e)e = sB \tag{12}$$

where

$$S_{ij} = \int_\Omega \nabla \times \vec{w}_{t,\,i} \cdot \frac{1}{\mu} \nabla \times \vec{w}_{t,\,j}\, dv \tag{13}$$

and Z, P_e and B are as defined in (6).

To prove the equivalence we need to show that the elimination of b from the system in (5) leads to an equation for e that is identical to (12). It is straightforward to show that this requires the proof of the following equality:

$$D^T P_b^{-1} D = S. \tag{14}$$

If μ is assumed to be constant or piecewise constant, the above relationship, written in a form that explicitly indicates the elements of the matrices, becomes

$$\begin{aligned} &\left[\int_\Omega \nabla \times \vec{w}_t \cdot \vec{w}_n\, dv\right]_{M\times N} \cdot \left[\int_\Omega \vec{w}_n \cdot \vec{w}_n\, dv\right]_{N\times N}^{-1} \\ &\cdot \left[\int_\Omega \vec{w}_n \cdot \nabla \times \vec{w}_t\, dv\right]_{N\times M} \\ &= \left[\int_\Omega \nabla \times \vec{w}_t \cdot \nabla \times \vec{w}_t\, dv\right]_{M\times M} \end{aligned} \tag{15}$$

where M and N are the dimensions of the vector spaces W_t and W_n, respectively.

It has been pointed out in [7] that Whitney-1 (W_1) and Whitney-2 (W_2) forms are related as

$$\nabla \times W_1 \subset W_2. \tag{16}$$

More specifically, W_1 is the lowest-order TV space containing the zeroth-order gradient and first-order nongradient components; W_2 is the lowest-order NV space containing the zeroth-order curl and first-order noncurl components. Because of (16), the bases in W_t and W_n are related

$$\begin{pmatrix} \nabla \times \vec{w}_{t,\,1} \\ \nabla \times \vec{w}_{t,\,2} \\ \vdots \\ \nabla \times \vec{w}_{t,\,M} \end{pmatrix} = T_{M\times N} \begin{pmatrix} \vec{w}_{n,\,1} \\ \vec{w}_{n,\,2} \\ \vdots \\ \vec{w}_{n,\,N} \end{pmatrix} \tag{17}$$

where $T_{M\times N}$ is the transition matrix between the two spaces. Substitution of (17) in the matrices on both sides of (15) leads directly to the desired result, namely the validity of the equality stated in (14).

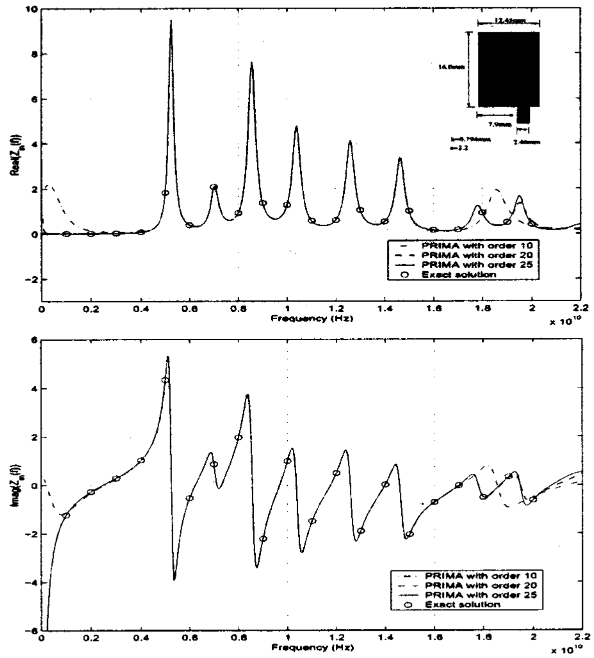

Fig. 1. Input impedance of a microstrip patch antenna. "Exact" solution refers to the response calculated one frequency point at a time from a direct solution of the vector wave equation.

The useful consequence of this equivalence for the purposes of Krylov model order reduction is that in the generation of the orthonormal bases of Krylov subspace $Kr(A, R, n)$, $(G + s_0 C)^{-1} X$ can be performed efficiently as a two-step process. Step 1 involves the calculation of the electric field vector of unknown e, and is based on the one-time LU factorization of the matrix $(S + s_0 Z + s_0^2 P_e)$. This matrix is much smaller than the matrix $(G + s_0 C)$ that would have to be factored otherwise. The second step involves the calculation of the magnetic flux vector of unknown b as follows:

$$\nabla \times \vec{E} = -s\vec{B} \rightarrow De = -sP_b b \rightarrow b = -\frac{1}{s} Te. \qquad (18)$$

IV. Numerical Results

To demonstrate the validity of the proposed formulation and the associated model order reduction through Krylov methods, we consider the broadband extraction of the electromagnetic response of a microstrip patch antenna. The dimensions of the antenna are shown in the insert of Fig. 1. First order ABC's are used at the top and side truncation boundaries of the domain. The distance of the ABC boundaries from the patch are 2.4 mm at the top and 4.5 mm at the sides. The representation of the fields on the microstrip port boundary is in terms of the microstrip modes. The objective of model order reduction is to generate directly a reduced-order model with the response valid over a broad frequency range. For the purposes of this numerical example, the bandwidth of interest is $0 < f < 22$ GHz, and the expansion frequency $s_0 = j2\pi \times 10$ GHz. It is at this frequency that the LU decomposition of the FEM matrix in (12) is performed. The number of edge (electric field) unknowns is 23 177, while the number of facet (magnetic flux) unknowns is 43 012. Thus, the proposed methodology requires the LU decomposition of a matrix of size 23 177 instead of one of size 66 189 that would be required otherwise.

The total CPU time for a single-frequency LU decomposition of the sparse FEM matrix of size 23 117 and the generation of a macromodel of order 25 requires less than 4 min on a Pentium III (500 MHz) PC with 256 MB of memory. As depicted in Fig. 1, the reduced-order model of order 25 provides for excellent agreement over the entire bandwidth. This method is much faster than AWE, since AWE would have to be performed at (at least) three expansion frequencies, one on either end of the frequency bandwidth of interest and one at the center of the bandwidth, in order to capture accurately the response over the entire bandwidth.

V. Conclusion

In conclusion, this paper has demonstrated how Krylov model order reduction algorithms can be used for fast frequency sweep of lossy electromagnetic systems in unbounded domains approximated through an FEM model. Even though the FEM discrete model is generated for the system of Maxwell's curl equations, it is shown that use of proper set of spaces for the expansion of the electric field and magnetic flux density makes the model equivalent to the one obtained from an FEM approximation of the vector wave equation for the electric field only. This equivalence is exploited to keep the computational cost of the generation of the Krylov subspace and the associated reduced-order model approximately equal to that required for the direct solution of the popular vector wave equation-based FEM model at a single frequency.

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